1 Given that $M = \dfrac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2}$

find the values of n for which $M = 2$

$$M = \frac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2}$$

$$2 = \frac{(2\times 3^2)^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{2^4 \times 3^2} \quad (1)$$

$$2^1 = \frac{2^{4n} \times 2^{3n^2-18n} \times 3^{8n} \times 3^{2-8n}}{2^4 \times 3^2}$$

$$2^1 = 2^{3n^2-14n-4} \times 3^{8n-8n+2-2}$$

$$2^1 = 2^{3n^2-14n-4} \times 1$$

$$1 = 3n^2 - 14n - 4$$

$$3n^2 - 14n - 5 = 0 \quad (1)$$

$$n = \frac{14 \pm \sqrt{(-14)^2 - 4(3)(-5)}}{2(3)} \quad (1)$$

$$= \frac{14 \pm \sqrt{256}}{6} \Rightarrow \frac{14 \pm 16}{6}$$

$$n = \frac{30}{6}, \; n = -\frac{2}{6}$$

$$= 5, \; -\tfrac{1}{3} \quad (1)$$

$-\frac{1}{3}, 5$

(Total for Question 1 is 5 marks)

2 Given that $150^x = 1$

(a) write down the value of x.

$x^0 = 1$

$x =$ 0 (1)

(1)

Given that $3^{-8} \div 3^{-6} = 3^n$

(b) find the value of n.

$\frac{3^{-8}}{3^{-6}} = 3^n$

$3^{(-8-(-6))} = 3^n$

$3^{-2} = 3^n$

$n = -2$

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$n =$ -2 (1)

(1)

(Total for Question 2 is 2 marks)

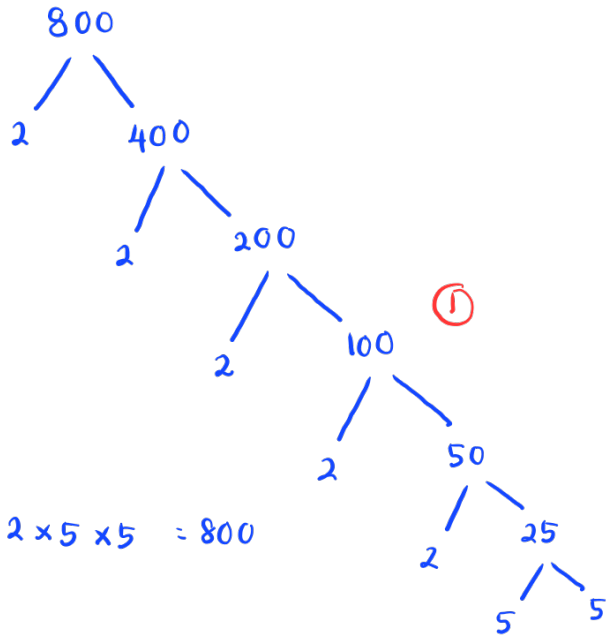
3 (a) Write $5^{17} \times 5^{2}$ as a single power of 5

$5^{17+2} = 5^{19}$

5^{19} (1)

(1)

(b) Write 800 as a product of its prime factors.
Show your working clearly.



(1)

$2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 800$

$2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$ (1)

(2)

(Total for Question 3 is 3 marks)

4 (c) Simplify $(p^2 + 3)^0$

$x^0 = 1$

1 (1)

.....

(1)

(Total for Question 4 is 1 marks)

5 (b) Simplify fully $\left(\frac{27a^{12}}{t^{15}}\right)^{-\frac{2}{3}}$

$\left(\frac{27a^{12}}{t^{15}}\right)^{-\frac{2}{3}} = \left(3^3 \times a^{12} \times t^{-15}\right)^{-\frac{2}{3}}$

$= (3^3)^{-\frac{2}{3}} \times (a^{12})^{-\frac{2}{3}} \times (t^{-15})^{-\frac{2}{3}}$ (1)

$= 3^{-2} \times a^{-8} \times t^{10}$ (1)

$= \frac{t^{10}}{9a^8}$ (1)

$\frac{t^{10}}{9a^8}$

(3)

(Total for Question 5 is 3 marks)

6 (a) Simplify $\frac{x^9}{x^2}$

$x^{9-2} = x^7$

x^7 (1)

(1)

(b) Write $\frac{7^8 \times 7^4}{7^3}$ as a single power of 7

$\frac{7^{8+4}}{7^3} = \frac{7^{12}}{7^3}$ (1)

$= 7^{12-3}$

$= 7^9$ (1)

7^9

(2)

(Total for Question 6 is 3 marks)

7 (b) Write down the value of g^0

1

.....

(1)

(Total for Question 7 is 1 marks)

8 (a) Write down the value of y^0

1 (1)

.....

(1)

(Total for Question 8 is 1 marks)

9 Simplify fully $\left(\frac{9t^4w^9}{18t^6w^{10}}\right)^{-2}$

$= \left(\frac{9t^4w^9}{18t^6w^{10}}\right)^{-2}$

$= \left(\frac{1}{2t^2w}\right)^{-2}$ (1)

$= (2^{-1}t^{-2}w^{-1})^{-2}$

$= 2^2t^4w^2$ (1)

$= 4t^4w^2$ (1)

$4t^4w^2$

(Total for Question 9 is 3 marks)

$P = 3^3 \times 5^2 \times 7$
$Q = 3^2 \times 5 \times 7^2$

10 (b) Work out the value of $P^3 \times Q$
Give your answer in the form $3^x \times 5^y \times 7^z$ where x, y and z are positive integers.

$P^3 = (3^3 \times 5^2 \times 7)^3$

$= 3^9 \times 5^6 \times 7^3$

$P^3 \times Q = (3^9 \times 5^6 \times 7^3) \times (3^2 \times 5 \times 7^2)$

$= 3^9 \times 3^2 \times 5^6 \times 5 \times 7^3 \times 7^2$

$= 3^{9+2} \times 5^{6+1} \times 7^{3+2}$

$= 3^{11} \times 5^7 \times 7^5$ (2)

$3^{11} \times 5^7 \times 7^5$

.....

(2)

(Total for Question 10 is 2 marks)

11 (b) Simplify fully $\left(\frac{2x^5}{8xy^2}\right)^{-2}$

Solve inside bracket first :

$\left[\frac{2x^5}{8xy^2}\right] = \left[\frac{x^4}{4y^2}\right]$

$\left[\frac{x^4}{4y^2}\right]^{-2}$ (1) $= (x^4)^{-2} \times (4^{-1})^{-2} \times (y^{-2})^{-2}$ (1)

$= x^{-8} \times 4^2 \times y^4$

$= \frac{16y^4}{x^8}$ (1)

$\frac{16y^4}{x^8}$

(3)

(Total for Question 11 is 3 marks)

12 (b) Simplify a^0 where $a > 0$

1 (1)

(1)

(c) Simplify fully $\frac{3xy^3}{6x^2y}$

$\frac{3}{6} \times \frac{x}{x^2} \times \frac{y^3}{y}$

$= \frac{1}{2} \times \frac{1}{x} \times y^2$

$= \frac{y^2}{2x}$ (2)

$\frac{y^2}{2x}$

(2)

(Total for Question 12 is 3 marks)

13 (a) Simplify $8 \times (4t)^0$

8 (1)

.....

(1)

$x^6 \div x^{-5} = x^p$

(b) Find the value of p

$6 - (-5) = 11$

$p =$ 11 (1)

.....

(1)

(c) Simplify fully $(2k^2m^4)^3$

$2^3 \times k^{2\times3} \times m^{4\times3}$ (1)

$= 8k^6m^{12}$ (1)

$8k^6m^{12}$

.....

(2)

(Total for Question 13 is 4 marks)

14 $$\frac{18 \times \left(\sqrt{27}\right)^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

Express x in terms of n
Show your working clearly and simplify your expression.

$$\frac{\cancel{2} \times \cancel{3} \times 3 \times \left(3^{\frac{3}{2}}\right)^{4n+6}}{\cancel{2} \times \cancel{3} \times 3^{2(2n+8)}} \quad (1)$$

$$\frac{3 \times 3^{6n+9}}{3^{4n+16}} = \frac{3^2 \times 3^{6n+9}}{3 \times 3^{4n+16}} = 3^x \quad (1)$$

$$= \frac{3^{6n+11}}{3^{4n+17}} = 3^x$$

$$3^{6n+11} = 3^x \times 3^{4n+17}$$

$$6n+11 = x + 4n + 17$$

$$x = 2n - 6 \quad (1)$$

$x =$ 2n - 6

(Total for Question 14 is 3 marks)

15 (a) Simplify $x^4 \times x^5$

$x^{4+5} = x^9$

x^9 (1)

(1)

(b) Simplify $(4y^2)^3$

$4^3 \times y^{2\times3}$ (1)

$= 64y^6$ (1)

$64y^6$

(2)

(c) Factorise $n^2 - 7n + 12$

$(n-3)(n-4)$ (2)

$(n-3)(n-4)$

(2)

(Total for Question 15 is 5 marks)

16 (a) Write down the value of x^0

1 (1)

.....

(1)

Given that $2^{-3} \times 2^9 = 2^n$

(b) find the value of n

$2^{-3+9} = 2^n$

$n = 6$

$n =$ 6 (1)

.....

(1)

Given that $\dfrac{7^{206} \times 7^m}{7^{214}} = 7^{-3}$

(c) find the value of m

$206 + m - 214 = -3$ (1)

$m - 8 = -3$

$m = 5$ (1)

$m =$ 5

.....

(2)

(Total for Question 16 is 4 marks)

17 (a) Simplify $a^7 \times a^4$

$a^{7+4} = a^{11}$

a^{11} (1)

(1)

(b) Simplify $w^{15} \div w^3$

$w^{15-3} = w^{12}$

w^{12} (1)

(1)

(c) Simplify $(8x^5y^3)^2$

$8^2 \times x^{5\times2} \times y^{3\times2}$

$= 64x^{10}y^6$ (2)

$64x^{10}y^6$

(2)

(d) Make t the subject of $c = t^3 - 8v$

$t^3 = c + 8v$ (1)

$t = \sqrt[3]{c+8v}$ (1)

$t = \sqrt[3]{c+8v}$

(2)

(Total for Question 17 is 6 marks)

18 (b) Simplify completely $\left(\frac{16w^8}{y^{20}}\right)^{-\frac{3}{4}}$

$\left(\frac{2w^2}{y^5}\right)^{-3} = \frac{2^{-3}w^{-6}}{y^{-15}}$ (1)

(1)

$= \frac{y^{15}}{8w^6}$ (1)

$\frac{y^{15}}{8w^6}$

(3)

(Total for Question 18 is 6 marks)

19 (a) Simplify $\frac{2}{y^0}$

$\frac{2}{1} = 2$

2 (1)

(1)

(b) Simplify fully $(16a^4)^{\frac{3}{4}}$

$16^{\frac{3}{4}} \times a^{4(\frac{3}{4})}$

$= 8 \times a^3$

$= 8a^3$

$8a^3$ (2)

(2)

(Total for Question 19 is 3 marks)

20 $3^{\frac{1}{2}} \times 3^{\frac{2}{5}} = 3^m$

(a) Work out the value of m

$3^{\frac{1\times5}{2\times5}+\frac{2\times2}{5\times2}}$

$3^{\frac{5}{10}+\frac{4}{10}}$

$= 3^{\frac{9}{10}}$ $m = \frac{9}{10}$

$m =$ $\frac{9}{10}$ (1)

(1)

$5^{-10} \div 5^{-4} = 5^n$

(b) Work out the value of n

$5^{-10-(-4)} = 5^{-6}$

$n = -6$

$n =$ -6 (1)

(1)

(Total for Question 20 is 2 marks)

21 (a) Write down the value of $(m + 2)^0$ where m is a positive integer.

1

.....

(1)

(Total for Question 21 is 1 marks)

22 Find the values of n such that

$$\frac{10^{4n} \times 2^{3(n^2-5n)} \times 5^{2(1-2n)}}{20^2} = 1$$

Show clear algebraic working.

$10^{4n} = (5 \times 2)^{4n}$

$20^2 = (5 \times 2^2)^2$ (1)

$$\frac{5^{4n} \times 2^{4n} \times 2^{3n^2-15n} \times 5^{2-4n}}{5^2 \times 2^4} = 1$$

$$= \frac{5^{4n+2-4n} \times 2^{4n+3n^2-15n}}{5^2 \times 2^4} = 1$$

$$= \frac{5^2 \times 2^{3n^2-11n}}{5^2 \times 2^4} = 1$$ (1)

$2^{3n^2-11n-4} = 2^0$ (1)

$3n^2 - 11n - 4 = 0$ (1)

$(3n+1)(n-4) = 0$

$n = -\frac{1}{3}, \; n = 4$ (1)

$-1/3, 4$

(Total for Question 22 is 5 marks)

23 (a) Simplify $m^{10} \div m^3$

$m^{10-3} = m^7$ (1)

m^7

.....

(1)

$k^n \times k^4 = k^{12}$

(b) Write down the value of n

$k^{n+4} = k^{12}$

$n + 4 = 12$

$n = 8$ (1)

$n =$ 8

.....

(1)

(c) Simplify $(3x^6y^8)^2$

$3^2 \times x^{6\times2} \times y^{8\times2}$

$= 9x^{12}y^{16}$ (2)

$9x^{12}y^{16}$

.....

(2)

(Total for Question 23 is 4 marks)

24 (a) Simplify $(4^{-2})^0$

1 (1)

.....

(1)

$3^{-14} \times 3^8 = 3^m$

(b) Find the value of m

$3^{-14+8} = 3^m$

$-14+8 = m$

$-6 = m$ (1)

$m =$ -6

.....

(1)

(Total for Question 24 is 2 marks)

25 (a) Simplify $(2c^4d^7)^3$

$2^3 \times c^{4\times3} \times d^{7\times3}$

$= 8c^{12}d^{21}$

$8c^{12}d^{21}$ (2)

(2)

(b) Find the value of $5y^0$ where $y > 0$

$y^0 = 1$

$5(1) = 5$ (1)

5

(1)

(c) Factorise fully $16a^2b^3 + 20a^3b$

$4(4a^2b^3 + 5a^3b)$

$4a^2(4b^3 + 5ab)$

$4a^2b(4b^2 + 5a)$ (2)

$4a^2b(4b^2+5a)$

(2)

(d) (i) Factorise $x^2 + 9x - 22$

$(x \pm 11)(x \pm 2)$ (1)

$= (x+11)(x-2)$ (1)

$(x+11)(x-2)$

(2)

(ii) Hence solve $x^2 + 9x - 22 = 0$

$-11, 2$ (1)

(1)

(Total for Question 25 is 8 marks)